Dronacharya Group of Institutions, Gr. Noida

Department of Applied Sciences (First Year)

Even Semester (2020-2021)

Objective Question Bank

Subject Name & Code: Engineering Mathematics-II (KAS 203T)

Unit No.& Unit Name: Unit II (<u>Multivariable calculus-II</u>)

PART-I(Beta & Gaama Function)

1. Which of the following is true?
   A) Γ(n+1) = nΓ(n) for any real number B) Γ(n) = nΓ(n+1) for any real number
   C) Γ(n+1) = nΓ(n) for n>1 D) Γ(n) = nΓ(n+1) for n>1
2. Γ(n+1) = n! can be used when ____________
   a) n is any integer b) n is a positive integer
   c) n is a negative integer d) n is any real number
3. Gamma function is said to be as Euler's integral of
   a) first kind b) 2nd kind
   (c)3rd Kind (d) None of these
4. What is the value of $\Gamma\left(\frac{1}{2}\right)$?

   a) $\sqrt{\pi}$ b) $\frac{1}{2}\sqrt{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{\sqrt{2}}$

5. What is the value of $\Gamma\left(\frac{7}{2}\right)$?

   a) $\frac{15}{8}\sqrt{\pi}$ b) $\frac{1}{2}\sqrt{\pi}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{\sqrt{2}}$

6. The value of $\Gamma\left(-\frac{1}{2}\right)$

a) $-\frac{8}{15}\sqrt{\pi}$ b) $\frac{4}{3}\sqrt{\pi}$ c) $-2\sqrt{\pi}$ d) $\sqrt{\pi}$

7.The value of $\Gamma\left(-\frac{3}{2}\right)$

a) $-\frac{8}{15}\sqrt{\pi}$ b) $\frac{4}{3}\sqrt{\pi}$ c) $-2\sqrt{\pi}$ d) $\sqrt{\pi}$

8.The value of $\Gamma\left(-\frac{5}{2}\right)$

a) $-\frac{8}{15}\sqrt{\pi}$ b) $\frac{4}{3}\sqrt{\pi}$ c) $-2\sqrt{\pi}$ d) $\sqrt{\pi}$

9. What is the value of $\int_0^{\infty} e^{-x^2} dx$

a) $\sqrt{\pi}$ b) $\sqrt{\pi}/\sqrt{2}$ c) $\frac{\sqrt{\pi}}{2}$ d) $\frac{\pi}{\sqrt{2}}$

10. What is the value of the integral $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$

a) $\Gamma(3/4)^2/\sqrt{\pi}$ b) $\Gamma(1/4)^2/\sqrt{\pi}$ c) $\Gamma(3/4)^2/\pi$ d) $\frac{\pi}{\sqrt{2}}$

11. What is the value of the integral $\int_0^{\infty} \frac{x^c}{c^x} dx$?

a) $\frac{\Gamma(c+1)}{(\log c)^c}$ b) $\frac{\Gamma(c+1)}{(\log c)^{c+1}}$ c) $\pi/\log c$ d) $1/2\log c$

12. What is the value of $\int_0^{\infty} \frac{1}{1+x^4} dx$

a) $\frac{\pi\sqrt{2}}{4}$ b) $\frac{\pi\sqrt{2}}{3}$ c) $\frac{3\pi\sqrt{2}}{4}$ d) $\sqrt{3}\pi/4$

13. The value of the integral $I = \int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}} dx$

a) $\sqrt{\pi}$ b) 0 c) $\frac{\sqrt{\pi}}{3}$ d) $\frac{3}{2}\sqrt{\pi}$

14. By Beta and Gamma function value of $\int_0^{\infty} x^{1/4} e^{-\sqrt{x}} dx$

a) $\sqrt{\pi}$ b) $\sqrt{\pi}/\sqrt{2}$ c) $\frac{\sqrt{\pi}}{2}$ d) $\frac{3}{2}\sqrt{\pi}$

15. The value of $\Gamma(n)\Gamma(1-n)$ is

(A) $\frac{\pi}{\cos\left(\frac{\pi}{2}-n\pi\right)}$ (B) $\frac{\pi}{\sin\left(\frac{\pi}{2}-n\pi\right)}$ (C) $\frac{\pi}{\sin(n\pi)}$ (D) Both (A) and (C)

16.The value of the $\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(\frac{3}{2}+p\right)$ is equal to

(A) $\left(\frac{1}{4}-p^2\right)\pi\sec p\pi$ (B) $\left(\frac{1}{4}-p^2\right)\sec p\pi$ (C) $\left(\frac{1}{4}-p\right)\pi\sec p\pi$ (D) None of these

17. The value of $\Gamma\left(\frac{1}{n}\right)\Gamma\left(1-\frac{1}{n}\right)$ is

a) $\frac{\pi}{\sin n\pi}$ **b)** $n\Gamma(n)$ c) $\frac{\pi}{\sin\left(\frac{\pi}{n}\right)}$ **d)** $\beta(n,n)\Gamma(2n)$

18. The value of $\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx$ will exist iff

(A) Only for $n > 0$ (B) Only for $n < 0$ (C) For every n (D) None of these

19. Consider the following Statements:

(i) $\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$ will exists iff $m>0\,\&\,n>0$.

(ii) $\beta(m,n)=\beta(n,m)$ (i.e. Beta function is symmetric)

(iii) $\beta(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$

(iv) $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Which of the following are correct?

(A) only (i)&(ii) (B) All of the above

(C) only (i),(ii)& (iii) (D) only (i)&(iv)

20. The value of $[\Gamma(n)]^2$

a) $\frac{\pi}{\sin n\pi}$ **b)** $n\Gamma(n)$ c) $\frac{\pi}{\sin(\frac{\pi}{n})}$ **d)** $\beta(n,n)\Gamma(2n)$

21. The value of the integral $\mathrm{I}=\int_0^1\left(\frac{x^3}{1-x^3}\right)^{1/2}dx$

a) $\frac{1}{2}\beta(\frac{7}{6},\frac{6}{5})$ b) $\frac{1}{3}\beta(\frac{1}{2},\frac{5}{6})$ c) $\frac{1}{2}\beta(\frac{5}{4},\frac{4}{3})$ d) $\frac{1}{3}\beta(\frac{3}{6},\frac{1}{2})$

22. What is the value of $\int_0^1\frac{x^2}{\sqrt{1-x^4}}dx$

a) $\frac{1}{4}\beta\left(\frac{3}{4},\frac{1}{2}\right)$ b) $\beta\left(\frac{1}{4},\frac{1}{2}\right)$ c) $\beta\left(\frac{3}{4},\frac{1}{2}\right)$ d) $\beta\left(\frac{1}{2},\frac{1}{2}\right)$

23.The value of $\int_0^1\sqrt{x}\sqrt[3]{(1-x)}dx$

a) $\beta(\frac{7}{6},\frac{6}{5})$ b) $\beta(\frac{5}{2},\frac{3}{2})$ c) $\beta(\frac{3}{2},\frac{4}{3})$ d) $\beta(\frac{3}{2},\frac{5}{2})$

24.The value of $\int_0^1\sqrt[3]{x}\sqrt[4]{(1-x)}dx$

a) $\beta(\frac{4}{3},\frac{5}{4})$ b) $\beta(\frac{5}{4},\frac{6}{5})$ c) $\beta(\frac{5}{4},\frac{4}{3})$ d) $\beta(\frac{3}{2},\frac{4}{3})$

25 .The value of $\int_0^1\sqrt[4]{x}\sqrt[5]{(1-x)}dx$

a) $\beta(\frac{7}{6},\frac{6}{5})$ b) $\beta(\frac{5}{4},\frac{6}{5})$ c) $\beta(\frac{5}{4},\frac{4}{3})$ d) $\beta(\frac{3}{2},\frac{4}{3})$

26) The value of $\frac{\beta(m+1,n)}{\beta(m,n)}$ is.......................

27) The value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ is.......................

28) If $\beta(m,3)$=1/60 and m is the positive integer, the value of m is

a) 4 b) 3 c)2 d) 5

29. The Value of the $\int_0^\infty e^{-x^4}dx$ is

(A) $\frac{1}{2}\sqrt{\pi}$ (B) $\frac{1}{4}\Gamma\left(\frac{3}{4}\right)$ (C) $\frac{1}{4}\Gamma\left(\frac{1}{4}\right)$ (D) $\frac{3}{2}\sqrt{\pi}$

30. The value of $\int_0^2 x\left(8-x^3\right)^{1/3}dx$ is

(A) $\frac{8}{3}\beta\left(\frac{1}{3},\frac{4}{3}\right)$ (B) $\frac{7}{3}\beta\left(\frac{2}{3},\frac{4}{3}\right)$

(C) $\frac{8}{3}\beta\left(\frac{2}{3},\frac{4}{3}\right)$ (D) $\frac{8}{3}\beta\left(\frac{2}{3},\frac{5}{3}\right)$

31.What is the value of $\int_0^1 \frac{1}{\sqrt{1-x^4}}dx$

a) $\beta\left(\frac{3}{4},\frac{1}{2}\right)$ b) $\frac{1}{4}\beta\left(\frac{1}{4},\frac{1}{2}\right)$ c) $\beta\left(\frac{2}{3},\frac{1}{2}\right)$ d) $\beta\left(\frac{1}{2},\frac{1}{2}\right)$

32. The value of the integral $\int_0^1 x^5\left(1-x^3\right)^3 dx$

a) $\frac{1}{590}$ b) $\frac{1}{60}$ c) $\frac{1}{396}$ d) $\frac{1}{386}$

PART-II (Dirichlet's Integration and Applications of Integratins)

33. The area in the first quadrant enclosed by the curve $\left(\frac{x}{a}\right)^m+\left(\frac{y}{b}\right)^n=1,$ where $m>0, n>0,$ is

(A) $\frac{mn}{ab}\frac{\Gamma\left(\frac{1}{m}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{m}+\frac{1}{n}+1\right)}$ (B) $\frac{ab}{mn}\frac{\Gamma\left(\frac{1}{a}\right)\Gamma\left(\frac{1}{b}\right)}{\Gamma\left(\frac{1}{a}+\frac{1}{b}+1\right)}$

(C) $\frac{mn}{ab}\frac{\Gamma\left(\frac{1}{a}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{m}+\frac{1}{n}+1\right)}$ (D) $\frac{ab}{mn}\frac{\Gamma\left(\frac{1}{m}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{m}+\frac{1}{n}+1\right)}$

34. The value of Dirichlet's Integral $\iiint_V x^{l-1}y^{m-1}z^{n-1}dxdydz;$ where V is bounded by $x\geq 0, y\geq 0, z\geq 0$ and $x+y+z\leq 1$ is

(A) $\frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n+1)}$ (B) $\frac{\Gamma\left(\frac{1}{m}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{m}+\frac{1}{n}+1\right)}$

(C) $\frac{\Gamma\left(\frac{1}{l}\right)\Gamma\left(\frac{1}{m}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{l}+\frac{1}{m}+\frac{1}{n}+1\right)}$ (D) $\frac{ab}{mn}\frac{\Gamma\left(\frac{1}{m}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{m}+\frac{1}{n}+1\right)}$

35. By using the Dirichlet's integral the volume of an ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,$ is

(A) $\frac{1}{3}\pi abc$ (B) $\frac{1}{6}\pi abc$ (C) $\frac{2}{3}\pi abc$ (D) $\frac{4}{3}\pi abc$

36. The Value of the integral $\iiint x^2 yz dx\, dy\, dz,$ where x, y, z are all positive $x \geq 0,\ y \geq,\ z \geq 0\ and\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is

a) $\frac{abc}{2520}$ b) $\frac{a^2bc}{2530}$ c) $\frac{a^2b^2c^2}{2520}$ d) $\frac{abc^2}{2520}$

37) The Value of the integral $\iiint x^2 dx\, dy\, dz,$ where x, y, z are all positive

$x \geq 0,\ y \geq,\ z \geq 0\ and\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

a) $\frac{a^2bc}{60}$ b) $\frac{a^2b^2c}{30}$ c) $\frac{abc^2}{60}$ d) $\frac{a^2b^2c^2}{90}$

38) Apply Dirichlet's Integral the mass of an octant of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,$ the density at any point being $\rho = k\ xyz$ is

(A) $\frac{a^2b^2c^2}{48}$ (B) $\frac{ka^2b^2c^2}{6}$ (C) $\frac{ka^2b^2c^2}{16}$ (D) $\frac{ka^2b^2c^2}{48}$

39) The volume of solid bounded by coordinate planes and surface $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}+\sqrt{\frac{z}{c}}=1,$

(A) $\frac{abc}{90}$ (B) $\frac{abc}{60}$ (C) $\frac{abc}{96}$ (D) $\frac{a^2b^2c^2}{96}$

40. The mass of the region in xy-plane bounded by $x=0, y=0, x+y=1$ with density $k\sqrt{xy}$ is given by

(A) $\frac{k\pi}{24}$ (B) $\frac{k\pi}{60}$ (C) $\frac{k\pi}{96}$ (D) $\frac{k\pi^2}{96}$

41.Consider the following Statements:

(i) The volume of solid of revolution about $x-$axis of the area bounded by curve $y = f(x)$, $x-$axis and lines $x = a$ and $x = b$ is given by $V = \int_a^b \pi y^2 dx$.

(ii) The volume of solid of revolution about $y-$axis of the area bounded by curve $x = f(y)$, $y-$axis and lines $y = c$ and $y = d$ is given by $V = \int_c^d \pi x^2 dy$.

Which of the above is correct? Choose the correct option

(A) Only(i) (B) only(ii) (C) Both (i) &(ii) (D) None of these

42.Consider the following Statements:

The volume of solid of revolution generated by revolving the plane area R, bounded by curve C whose equation is given in polar form $r = f(\theta)$ and radii vectors $\theta = \theta_1, \theta = \theta_2$

(i) About the initial line OX $(\theta = 0)$ is $V = \frac{2\pi}{3}\int_{\theta_1}^{\theta_2} r^3 \sin\theta\, d\theta$

(ii) About the initial line through pole and perpendicular to initial line, i.e., OY $\left(\theta = \frac{\pi}{2}\right)$ is $V = \frac{2\pi}{3}\int_{\theta_1}^{\theta_2} r^3 \cos\theta \, d\theta$.

Which of the above is correct? Choose the correct option

(A) Only(i) (B) only(ii) (C) Both (i) &(ii) (D) None of these

43.Consider the following Statements:

(i) The Area of surface of solid of revolution of generated by revolving the arc AB of the curve $y = f(x)$, $x-$axis and lines $x = a$ and $x = b$ is given by $S = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$.

(ii) The Area of surface of solid of revolution of generated by revolving the arc CD of the curve $x = g(x)$, $y-$axis and lines $y = c$ and $y = d$ is given by $S = \int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$.

Which of the above is correct? Choose the correct option

(A) Only(i) (B) only(ii) (C) Both (i) &(ii) (D) None of these

44. The parabolic arc $y = \sqrt{x}$, $1 \le x \le 2$ is revolved about $x-$ axis. Then the volume of solid of revolution is

a) $\frac{3\pi}{4}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{2}$ d) none of these

45. The circle $x^2 + y^2 = a^2$ is revolved about $x-$ axis. Then the area of surface of revolution is

a) $4\pi a$ b) πa^2 c) $4\pi a^2$ d) $2\pi a$

46. The area of the surface generated by rotating about x-axis the arc of the curve $y = x^3$ between x=0 and x=1 is....................................

47. The area of the surface generated by rotating about x-axis the arc of the curve $y = \sin x$ between x=0 and x=π is...................................

48. Consider the improper integrals

(i) $\int_1^\infty \frac{dx}{\sqrt{x}}$ (ii) $\int_{-\infty}^0 e^{2x}dx$ (iii) $\int_{-\infty}^\infty \frac{dx}{x^2+2x+2}$ (iv) $\int_0^\infty \frac{e^{-x}}{\sqrt{x}}dx$ (v) $\int_0^1 \frac{dx}{x^2}$

Which of the following is/are improper integral of first kind?

(A) Only (i),(ii)&(iii) (B) Only (i), (iv) &(v) (C) Only (i)&(ii) (D) All

49. Consider the improper integrals

(i) $\int_1^\infty \frac{dx}{\sqrt{x}}$ (ii) $\int_1^2 \frac{1}{2-x}dx$ (iii) $\int_{-\infty}^\infty \frac{dx}{x^2+2x+2}$ (iv) $\int_0^\infty \frac{e^{-x}}{\sqrt{x}}dx$ (v) $\int_1^4 \frac{dx}{(x-1)(4-x)}$

(vi) $\int_0^1 \frac{dx}{x^2}$

Which of the following is/are improper integral of second kind?

(A) Only (ii),(v)&(vi) (B) Only (i)&(iii) (C) Only (ii)&(ii) (D) All

50. Consider the improper integrals

(i) $\int_1^{\infty} \frac{dx}{\sqrt{x}}$ (ii) $\int_1^{2} \frac{1}{2-x}dx$ (iii) $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$ (iv) $\int_0^{\infty} \frac{e^{-x}}{\sqrt{x}}dx$ (v) $\int_1^{4} \frac{dx}{(x-1)(4-x)}$

(vi) $\int_0^{1} \frac{dx}{x^2}$

Which of the following is/are improper integral of third kind?

(A) Only (ii),(v)&(vi) (B) Only (iv) (C) Only (ii)&(ii) (D) All

51. The value of p-integral $\int_0^{\infty} \frac{1}{x^p}dx$ converges if

(A) only $p>1$ (B) only $p<1$ (C) only $p=1$ (D) Both(B) and (C)

52. The value of p-integral $\int_a^{b} \frac{1}{(x-a)^p}dx$ OR $\int_a^{b} \frac{1}{(b-x)^p}dx$ converges if

(A) only $p>1$ (B) only $p<1$ (C) only $p=1$ (D) Both(B) and (C)

53. (First Comparison Test): If $0 \le f(x) \le g(x)$ for all x.

(i) $\int g(x)dx$ converges $\Rightarrow \int f(x)dx$ converges

(ii) $\int f(x)dx$ diverges $\Rightarrow \int g(x)dx$ diverges

Which of the above series is/are correct? Choose the correct option

(A) Only (i) (B) Only (ii)

(C) Both (i) and (ii) (D) None of these

Q49. (Limit form Comparison Test): If $f(x)$ & $g(x)$ be two positive functions on $[a,\infty]$ such that $\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right) = l(\ l \ne 0$ and finite$)$. Then, choose the correct option

(A) $\int_a^{\infty} f(x)dx$ & $\int_a^{\infty} g(x)dx$ behave alike

(B) Both $\int_a^{\infty} f(x)dx$ & $\int_a^{\infty} g(x)dx$ converge or diverge together

(C) $\int_a^{\infty} f(x)dx$ converge but $\int_a^{\infty} g(x)dx$ diverge

(D) Both (A) & (B)

Q50. For a function $f(x)$ which changes its sign and if $\int_a^{\infty} |f(x)|dx$ converges then $\int_a^{\infty} f(x)dx$

(A) Converges (B) Absolute converges (C) Diverges (D) Both (A) & (B)

51. The Improper Integral $\int_1^{\infty} \frac{\sin 2x}{x^5}dx$ converges to

(A) 1/4 (B)1/5 (C) -1/4 (D) None of These

52. The following improper integral $\int_0^{1/e} \frac{1}{x(\log x)^2}dx$

a)0 b) -1 c) 1 d) diverges to ∞

53. The following improper integral $\int_{1}^{2} \frac{1}{x(\log x)} dx$

a)0 b) -1 c) 1 d) diverges to ∞

54. The following improper integral $\int_{-2}^{2} \frac{1}{x+1} dx$

a) 0 (b) ½ ln3 (c) divergent (d) 8/9

55. The value improper integral $\int_{2}^{\infty} xe^{-x} dx$

a) – 2/ e^2 (b) 1 /e^2 (c) divergent (d) 3 /e^2

56. The following improper integral $\int_{-3}^{3} \frac{1}{(x+2)^3} dx$

a) 12/25 b) 0 c) 13/25 d) diverges to ∞

57. The following improper integral $\int_{-1}^{1} \frac{1}{x^2} dx$

a) 0 b) -1 c) 1 d) diverges to ∞

58.The following improper integral $\int_{0}^{1} \frac{1}{x(\log x)} dx$

a) converges to -1/4 b) converges to 1/4
c) converges to 1/2 d) diverges to ∞

59. The following improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

a) converges to π/2 b) converges to π c) converges to –π/2 d) none of these

60.The following improper integral $\int_{-\infty}^{\infty} e^{-x} dx$

a) 0 b) -1 c) 1 d) diverges to ∞

61. The value of improper integral $\int_{1}^{\infty} \frac{1}{x^{3/2}} dx$

a) 0 b) -1 c) 2 d)- 2

62. The value of following improper integral $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$

a) convergent b) divergent c) converges to π d) none of these

63. The value of following improper integral $\int_{0}^{2} \frac{dx}{\sqrt{4-x^2}}$

a) converges to π b) converges to π/2 c) converges to -π d) none of these

64. The Improper Integrals $\int_{0}^{\infty} \frac{dx}{a^2+x^2} dx$, if they exists the converges to

(A) $\frac{\pi}{2a}$ (B) $\frac{1}{2a}$ (C) None of These (B) $\frac{\pi}{2}$

65. The Improper Integrals $\int_0^\infty x \sin x dx$, if they exists then its value is

(A) $\frac{\pi}{2a}$ (B) $\frac{1}{2a}$ (C) None of These (B) $\frac{\pi}{2}$

Dronacharya Group of Institutions, Gr. Noida

Department of Applied Sciences (First Year)

Even Semester (2020-2021)

Objective Question Bank

Subject Name & Code: Engineering Mathematics-II (KAS 203T)

Unit No.& Unit Name: Unit III (Fourier Series & Sequence & Series)

PART-1 (FOURIER SERIES)

Q1. The period of the function $f(x) = \sin x + \frac{1}{2}\cos 2x + \frac{2}{3}\cos 3x$ is

(A) 2π (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{4\pi}{3}$

Q2. If T_1 and T_2 are periods of $f(x)$ and $g(x)$, then the period of $af(x)+bg(x)$ is the

(A) H.C.M. $\{T_1, T_2\}$ (B) L.C.M. $\{T_1, T_2\}$

(C) L.C.M. $\{T_2, T_1\}$ (D) Both (B)&(C)

Q3. In the Fourier series $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right\}$ for $c < x < c+2l$. Then the values of a_0, a_n and b_n are known as by

(A) Fourier's formulae (B)Euler's formulae

(C) Dirichlet's formulae (D) None of these

Q4. In the Fourier series $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right\}$ for $c < x < c+2l$. The constants a_0, a_n and b_n are called

(A) Fourier's Coefficient's (B)Euler's Coefficient's

(C) Dirichlet's Coefficient's (D) None of these

Q5. Any function $f(x)$ can be expressed as a Fourier series

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)\right\}$ for $c < x < c+2l$, where a_0, a_n and b_n are constants, provided

(i) $f(x)$ is periodic, single valued and finite.

(ii) $f(x)$ has a finite number of finite discontinuities in any one period.

(iii) $f(x)$ has finite number of maxima and minima.

Then all above conditions are known as by

(A) Fourier's conditions

(B)Euler's conditions

(C) Dirichlet's conditions

(D) None of these

Q6. In the Fourier series representation for the function $f(x)=\frac{1}{4}(\pi-x)^2$ in the interval $(0, 2\pi)$. The value of a_0 is

(A) $\frac{\pi^2}{6}$

(B) $\frac{\pi^2}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{2\pi^2}{3}$

Q7. The value of constant term if the function $f(x)=x+x^2$ is expanded in Fourier series defined in (-1,1) is given by

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $-\frac{4}{3}$

Q8. If $f(x)=x\sin x$ in $(-\pi,\pi)$ then the value of b_n is

(A) $\frac{2\pi}{3}$

(B) $\frac{1}{3}$

(C) $-\frac{\pi}{6}$

(D) 0

Q9. If $f(x)=x^2$ in $(-2,2)$ and $f(x+4)=f(x)$, then the value of a_n is

(A) $\frac{1}{2}\int_{-2}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right)dx$

(B) $\int_{0}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right)dx$

(C) $\frac{1}{2}\int_{0}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right)dx$

(D) Both (A) & (B)

Q10. If $f(x)=x\cos x$ in $(-3,3)$ then the value of a_0 is

(A) 0

(B) $-\frac{1}{3}$

(C) -1

(D) $\frac{2}{3}$

Q11. If $f(x)=x$ is expanded in Fourier sine series in $(0,\pi)$ then the value of b_n is

(A) $-\frac{1}{n}(-1)^n$

(B) $-\frac{2}{n}$

(C) $-\frac{2}{n}(-1)^n$

(D) $\frac{2}{n}(-1)^n$

Q12. If $f(x)=\begin{cases} x, & -\pi<x<0 \\ -x, & 0<x<\pi \end{cases}$, Then the value of $f(0)$ is

(A) 0

(B) $\frac{\pi}{2}$

(C) -2 (D) $\frac{\pi}{3}$

Q13. If $f(x)=1$, is expanded in a Fourier sine series in $(0,\pi)$ then the value of b_n is

(A) $\frac{2}{\pi n}\left\{1+(-1)^n\right\}$ (B) $\frac{2}{\pi n}\left\{1-(-1)^n\right\}$

(C) $\frac{1}{\pi n}\left\{1-(-1)^n\right\}$ (D) $-\frac{2}{\pi n}\left\{1-(-1)^n\right\}$

Q14. Half range Fourier sine series for the function $f(x)=x,\ 0<x<2$ is given by

(A) $x=-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2}$ (B) $x=-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2}$

(C) $x=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2}$ (D) $x=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2}$

Q15. The Fourier coefficient a_0 in Fourier series expansion of a function represents

(A) Always even function (B) Mean Value of the function

(C) Only odd function (D) None of these

Q16. If $f(x)=|x|,\ -\pi<x<\pi$, then the values of a_0 and b_n are

(A) $\pi,0$ (B) $0,\pi$

(C) $0,0$ (D) $0,-\pi$

Q17. If $f(x)=x^2,\ -\pi<x<\pi$, then the values of a_0 and b_n are

(A) $\frac{2\pi^2}{3},0$ (B) $-\frac{\pi^2}{3},0$

(C) $0,0$ (D) $\frac{4\pi}{3},0$

Q18. If we expand the function $f(x)=x\sin x$ as a Fourier series in the interval $-\pi\le x\le\pi$. Then the value of a_0 is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ (D) 2

Q19. If $f(x)=\begin{cases}0\ , & -\pi\le x\le 0\\ \sin x, & 0\le x\le\pi\end{cases}$, Then the value of a_0 is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $\frac{2}{\pi}$ (D) $\frac{4\pi}{3}$

Q20. If $f(x)=\begin{cases}x\ , & -\pi<x<0\\ -x, & 0<x<\pi\end{cases}$, Then the value of a_0 is

(A) $\frac{2\pi}{3}$ (B) $-\pi$

(C) $\frac{\pi}{6}$ (D) $\frac{4\pi}{3}$

Q21. If $f(x)=\frac{\pi-x}{2}$ for $0<x<2$, Then the value of a_n is

(A) $\frac{2}{3}$ (B) $\pi-1$

(C) $\frac{\pi}{6}$ (D) 0

Q22. If $f(x)=x-x^2$, $-1<x<1$, Then the value of a_0 is

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{4}{3}$

Q23. If $f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$, Then the value of a_0 is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$ (D) π

Q24. Half range Fourier cosine series for the function $f(x)=x,\ 0<x<2$ is given by

(A) $x=-\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cos\frac{n\pi x}{2}$ (B) $x=1-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cos\frac{n\pi x}{2}$

(C) $x=1+\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(\cos n\pi-1)}{n^2}\cos\frac{n\pi x}{2}$ (D) $x=1+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cos\frac{n\pi x}{2}$

Q25. The half-range sine series of $f(x)=t-t^2$ for $0<t<1$ is given by

(A) $\frac{8}{\pi^3}\left[\sin\pi t+\frac{1}{3^3}\sin 3\pi t+\frac{1}{5^3}\sin 5\pi t+\ldots\right]$

(B) $\frac{8}{\pi^3}\left[\sin 3\pi t+\frac{1}{3^3}\sin 5\pi t+\frac{1}{5^3}\sin 7\pi t+\ldots\right]$

(C) $\frac{8}{\pi^2}\left[\sin 3\pi t+\frac{1}{3^3}\sin 5\pi t+\frac{1}{5^3}\sin 7\pi t+\ldots\right]$

(D) $\frac{8}{\pi^2}\left[\sin\pi t+\frac{1}{3^3}\sin 3\pi t+\frac{1}{5^3}\sin 5\pi t+\ldots\right]$

Q26. In Fourier series expansion, if f(x) is ODD then ____________

(A) $a_0=0$, $a_n=0$ (B) $a_0\ne 0$, $a_n=0$

(C) $a_0=0$, $a_n\ne 0$ (D) $a_0\ne 0$, $a_n\ne 0$

Q27. In half range cosine series the value of a_0 is for $(0,\pi)$ is

(A) $\frac{2}{\pi}\int_0^{\pi} f(x)dx$ (B) $\frac{1}{\pi}\int_0^{\pi} f(x)dx$ (C) $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ (D) none of these

Q28. The trigonometric Fourier series of an even function does not have
(A) constant (B)cosine terms (C) sine terms (D) None of these

Q29. If $f(x)$ is discontinuous at x then the Fourier series converges to ________ where $f(x^+)$, $f(x^-)$ are respectively right hand and left hand limits of $f(x)$

(A) $\frac{f(x^+)+f(x^-)}{2}$ (B) $\frac{f(x^+)-f(x^-)}{2}$
(C) $\frac{f(x^+)+f(x^-)}{-2}$ (D) $\frac{f(x^+)-f(x^-)}{-2}$

Q30. A function $f(x) = \sin 2x + \cos 3x$ is _________ function in the interval $(-l, l)$.
(A) odd (B) $even$ (C) $neither\ even\ nor\ odd$ (D) $None\ of\ these$

Part-II (Sequence and Series)

Q31. Which of the following sequences is/are bounded ?
(i) $\left\langle \frac{1}{n} \right\rangle$ (ii) $\left\langle 1+(-1)^n \right\rangle$ (iii) $\left\langle (-1)^n \right\rangle$ (iv) $\left\langle \frac{1}{2^{n-1}} \right\rangle$
Choose the correct option
(A) only (i) (B) only (i) & (iii)
(C) only (i),(iii) & (iv) (D) All

Q32. A sequence $\langle a_n \rangle$ defined by $a_n = c, \forall n \in$ □ is called a
(A) Null sequence (B) Oscillating sequence
(C) Bounded Sequence (D) constant sequence

Q33. Which of the following sequences is/are monotonic ?
(i) $\left\langle \frac{1}{n} \right\rangle$ (ii) $\left\langle 1+(-1)^n \right\rangle$ (iii) $\left\langle (-1)^n \right\rangle$ (iv) $\left\langle \frac{1}{2^{n-1}} \right\rangle$ (v) $\left\langle 3^n \right\rangle$ (vi) $\left\langle \log n \right\rangle$
Choose the correct option
(A) only (i), (iv), (v) & (vi) (B) only (ii)& (iii)
(C) only (i),(iii) & (iv) (D) All

Q34. Which of the following sequences is/are convergent ?
(i) $\left\langle \frac{1}{n^2} \right\rangle$ (ii) $\left\langle 1+(-1)^n \right\rangle$ (iii) $\left\langle (-1)^n \right\rangle$ (iv) $\left\langle \frac{1}{2^{n-1}} \right\rangle$ (v) $\left\langle 3^n \right\rangle$ (vi) $\left\langle \log n \right\rangle$
(vii) $\left\langle n^2(-1)^n \right\rangle$
Choose the correct option
(A) only (i), (iv), (v) & (vi) (B) only (i), (iv)
(C) only (i),(iii) & (iv) (D) All

Q35. Which of the following sequences is/are divergent ?
(i) $\left\langle n+\frac{1}{n^2} \right\rangle$ (ii) $\left\langle 1+(-1)^n \right\rangle$ (iii) $\left\langle (-1)^n \right\rangle$ (iv) $\left\langle \frac{1}{2^{n-1}} \right\rangle$ (v) $\left\langle 3^n \right\rangle$ (vi) $\left\langle \log n \right\rangle$
(vii) $\left\langle n^2(-1)^n \right\rangle$
Choose the correct option
(A) only (i), (iv), (v) & (vi) (B) only (i) (v) & (vi)
(C) only (i),(iii) & (iv) (D) All

Q36. Which of the following sequences is/are oscillating finitely ?

(i) $\left\langle 1+\frac{1}{n^2} \right\rangle$ (ii) $\left\langle 2+(-1)^n \right\rangle$ (iii) $\left\langle \frac{1}{n}+(-1)^n \right\rangle$ (iv) $\left\langle \frac{1}{2^{n-1}} \right\rangle$ (v) $\langle \sin n \rangle$

(vi) $\langle \log n \rangle$ (vii) $\left\langle n^2 (-1)^n \right\rangle$

Choose the correct option

(A) only (i), (iv), (v) & (vi) (B) only (ii), (ii) & (v)

(C) only (i),(iii) & (iv) (D) All

Q37. Which of the following sequences is/are oscillating infinitely ?

(i) $\left\langle -1+\frac{1}{n^2} \right\rangle$ (ii) $\left\langle 2+(-1)^n \right\rangle$ (iii) $\left\langle \frac{1}{n}+(-1)^n \right\rangle$ (iv) $\left\langle 1-\frac{(-1)^n}{n} \right\rangle$ (v) $\langle 3^n \rangle$

(vi) $\left\langle (-1)^n \log n \right\rangle$ (vii) $\left\langle n^2 (-1)^n \right\rangle$

Choose the correct option

(A) only (i), (iv), (v) & (vi) (B) only (ii) & (ii)

(C) only (vi) & (vii) (D) All

Q38.Consider the following statements:

(i) Every convergent sequence has a unique limit.

(ii) Every convergent sequence is bounded but converse is not true.

(iii) A bounded monotonic sequence is convergent.

Choose which of the above is correct

(A) only (i) & (ii) (B) only (i) & (iii)

(C) only(iii) (D) All

Q39. Which of the following sequences is/are limits ?

(i) $\lim_{n\to\infty} \frac{1}{n^p} = 0, p > 0.$

(ii) $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$

(iii) $\lim_{n\to\infty} \frac{\log n}{n} = 0$

(iv) $\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x, \forall x$

(v) $\lim_{n\to\infty} x^{\frac{1}{n}} = 1,\ x > 0$

(vi) $\lim_{n\to\infty} x^n = 0,\ |x| < 1$

Choose the correct option

(A) only (i), (iv), (v) & (vi) (B) only (ii) & (ii)

(C) only (vi) & (vi) (D) All

Q40. Consider the following sequences are whose n^{th} term a_n are

(i) $a_n = \frac{n^2-1}{2n^2+n}$ (ii) $a_n = \tanh n$ (iii) $a_n = e^{n(-1)^n}$

Then sequence (i) converges to limit 1/2, (ii) converges to limit 1 and (iii) oscillating infinitely.

Choose the correct option

(A) only (ii) (B) only (iii)

(C) only (i) (D) All

Part-III (Infinite series)

Q41. An infinite series $\sum_{n=1}^{\infty} u_n$ converges or diverges or oscillates(finitely/infinitely) if and only if it's sequence of partial sums $\langle S_n \rangle$ is

(A) converges only
(B) converges or diverges only
(C) converges or diverges or oscillates(finitely/infinitely)
(D) None of these

Q42. Consider the following series:

(i) $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+....................\infty$

(ii) $1^2+2^2+............+n^2+.............\infty$

(iii) $7-4-3+7-4-3+7-4-3+............................\infty$

(iv) $\sum_{n=1}^{\infty}(-1)^{n-1} n$

Then series (i) converges and its sum=4/3 (ii) Divergent (iii) oscillates finitely (iv) oscillates infinitely. Choose the correct option

(A) only (i)
(B) only (i), (ii) &(iv)
(C) only (ii) and (iii)
(D) All (i),(ii),(iii)&(iv)

Q43. The necessary condition for the series $\sum u_n$ converges if

(A) $\lim_{n\to\infty} u_n = 0$
(B) $\lim_{n\to\infty} nu_n = 0$
(C) $\lim_{n\to\infty} u_n \neq 0$
(D) None of these

Q44. For a series $\sum u_n$ converges if $\lim_{n\to\infty} u_n \neq 0$ then

(A) $\sum u_n$ is not convergent
(B) $\sum u_n$ is convergent
(C) $\sum u_n$ may or may not be convergent
(D) None of these

Q45. Consider the following series:

(i) $1+\frac{2}{3}+\frac{3}{4}+...+\frac{n}{n-1}....................\infty$

(ii) $1+\frac{3}{5}+\frac{8}{10}+\frac{15}{17}...+\frac{2^n-1}{2^n+1}+....................\infty$

(iii) $\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{6}}+\sqrt{\frac{3}{8}}+.......+\sqrt{\frac{n}{2(n+1)}}+...........\infty$

(iv) $\sum_{n=1}^{\infty}\cos\frac{1}{n}$

Which of the above series is convergent? Choose the correct option

(A) only (i)
(B) only (i), (ii) &(iv)
(C) only (ii) and (iii)
(D) None of the above

Q46. The Geometrical series $\sum r^n = 1+r+r^2+.......+r^n+..............\infty$ is

(i) Convergent only if $|r|<1$

(ii) Divergent only if $r \geq 1$

(iii) Oscillates only if $r \leq -1$

Which of the above series is/are correct? Choose the correct option

(A) only (i) (B) only (i), (ii) &(iv)

(C) only (ii) and (iii) (D) ALL

Q47. The Harmonic series of order p or p -Harmonic series or p -series

$$\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+......+\frac{1}{n^p}+...........\infty$$

(i) Convergent only if $p>1$

(ii) Divergent only if $p\leq 1$

Which of the above series is/are correct? Choose the correct option

(A) Only (i) (B) Only (ii)

(C) Both (i) and (ii) (D) None of these

Q48. (First Comparison Test): If $\sum u_n$ & $\sum v_n$ be two positive term series such that $u_n \leq v_n \ \forall n \in \mathbb{N}$. Then

(i) $\sum v_n$ converges $\Rightarrow \sum u_n$ converges

(ii) $\sum v_n$ diverges $\Rightarrow \sum u_n$ diverges

Which of the above series is/are correct? Choose the correct option

(A) Only (i) (B) Only (ii)

(C) Both (i) and (ii) (D) None of these

Q49. (Limit form Comparison Test): If $\sum u_n$ & $\sum v_n$ be two positive term series such that $\lim_{n\to\infty}\left(\frac{u_n}{v_n}\right)=l\left(\ l\neq 0 \text{ and finite}\right)$. Then, choose the correct option

(A) $\sum u_n$ & $\sum v_n$ behave alike (B) Both $\sum u_n$ & $\sum v_n$ converge or diverge together

(C) $\sum u_n$ converge but $\sum v_n$ diverge (D) Both (A) & (B)

Q50. Consider the following series:

(i) $1+\frac{1}{2^2}+\frac{1}{3^3}+...+\frac{1}{n^n}$...................$\infty$

(ii) $\sum e^{-n^2}$

(iii) $\sum \frac{1}{\lfloor n}$

(iv) $\sum_{n=1}^{\infty}\frac{1}{n2^n}$

(v) $\sum \frac{1}{\sqrt{n}}$

Which of the above series is convergent? Choose the correct option

(A) only (i) (B) only (i), (ii) &(iv)

(C) only (ii) and (iii) (D)only (i),(ii),(iii)&(iv)

Q51. Consider the following series:

(i) $\sum\left(\frac{2^n+3}{3^n+1}\right)^{\frac{1}{2}}$

(ii) $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1}-n\right)$

(iii) $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$

Which of the above series is convergent? Choose the correct option

(A) only (i) (B) All (i), (ii) &(iii)

(C) only (ii) and (iii) (D)only (i) & (iii)

Q52. (D'Alembert Ratio Test): If $\sum u_n$ is a positive term series such that $\lim_{n\to\infty}\left(\frac{u_{n+1}}{u_n}\right)=l$. then $\sum u_n$ is

(i) convergent if $l<1$

(ii) divergent if $l>1$ and

(iii) test fails if $l=1$.

Which of the above is correct? Choose the correct option

(A) only (i) (B) All (i), (ii) &(iii)

(C) only (ii) and (iii) (D)only (i) & (iii)

Q53. Consider the following series: $\frac{1}{1.2}+\frac{1}{2.2^2}+\frac{1}{3.2^3}+\ldots\ldots\ldots\ldots\infty$

Choose the correct option

(A) convergent (B) Divergent

(C) Oscillates (D) None of these

Q54. (Raabe's Test or Higher Ratio Test): If $\sum u_n$ is a positive term series such that $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=l$. Then $\sum u_n$ is

(i) convergent if $l>1$

(ii) divergent if $l<1$ and

(iii) test fails if $l=1$.

Which of the above is correct? Choose the correct option

(A) only (i) (B) All (i), (ii) &(iii)

(C) only (ii) and (iii) (D)only (i) & (iii)

Q55. Consider the following series: $1+\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+\ldots\ldots\ldots\ldots\infty$

Choose the correct option

(A) convergent (B) Divergent

(C) Oscillates (D) None of these